

Chapter 6 Review *key*

Name _____ Date _____ Period _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question. **SHOW WORK FOR ALL PROBLEMS!**

Write the sum or difference in the standard form $a + bi$.

1) $(9 - 5i) + (8 + 9i)$
 $(9+8) + (-5i+9i)$
 $\boxed{17 + 4i}$

1) _____

2) $(4 + 5i) - (-2 + i)$
 $(4 - (-2)) + (5i - i)$
 $\boxed{6 + 4i}$

2) _____

Write the product in standard form.

3) $(\sqrt{15} + 9i)(\sqrt{15} - 9i)$
 Conjugates so, $(\sqrt{15})^2 + (9)^2 = 15 + 81 = \boxed{96}$

3) _____

Write the expression in standard form.

4) $\frac{(9 + 4i)}{(4 - 6i)} \cdot \frac{(4 + 6i)}{(4 + 6i)} = \frac{36 + 54i + 16i + 24i^2}{4^2 + 6^2} = \frac{36 + 70i - 24}{16 + 36}$
 $= \frac{12 + 70i}{52} = \boxed{\frac{3}{13} + \frac{35}{26}i}$

4) _____

Find the product of the complex number and its conjugate.

5) $(-2 - 5i)(-2 + 5i) = (-2)^2 + (5)^2 = 4 + 25 = \boxed{29}$

5) _____

Write the expression in the form $a + bi$, where a and b are real numbers.

6) $\frac{\sqrt{-20} \cdot \sqrt{-4}}{\sqrt{5}} = \frac{2\sqrt{5}i \cdot 2i}{\sqrt{5}} = \frac{4\sqrt{5}i^2}{\sqrt{5}} = \boxed{-4}$

6) _____

Find the absolute value of the complex number. Round your answer to two decimal places, if necessary.

7) $2 - 4i$
 $|2 - 4i| = \sqrt{(2)^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = \boxed{4.47}$

7) _____

Write the complex number in trigonometric form, using degree measure for the argument.

8) $6 + 8i$
In 1st Quadrant!
 $r = \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$
 $\theta = \tan^{-1}(8/6) = 53.1^\circ$
 $\boxed{10(\cos 53.1^\circ + i \sin 53.1^\circ)}$

8) _____

Write the complex number in the form $a + bi$.

9) $\sqrt{6}(\cos 315^\circ + i \sin 315^\circ)$

$$a = \sqrt{6} \cos 315^\circ = \sqrt{6} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$b = \sqrt{6} \sin 315^\circ = \sqrt{6} \cdot \left(-\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{12}}{2} = -\frac{2\sqrt{3}}{2} = -\sqrt{3}$$

9) $\boxed{\sqrt{3} - \sqrt{3}i}$

Perform the indicated operation. Write the answer in the form $a + bi$.

10) $4(\cos 135^\circ + i \sin 135^\circ) \cdot 6(\cos 225^\circ + i \sin 225^\circ)$

$$4 \cdot 6 (\cos(135^\circ + 225^\circ) + i \sin(135^\circ + 225^\circ))$$

$$24 (\cos(360^\circ) + i \sin(360^\circ)) = 24(1 + 0i) = \boxed{24}$$

10) _____

11) $\frac{8(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})}{3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})}$

$$\frac{8}{3} (\cos(\frac{\pi}{2} - \frac{\pi}{6}) + i \sin(\frac{\pi}{2} - \frac{\pi}{6}))$$

$$\frac{8}{3} (\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3})) = \frac{8}{3} (\frac{1}{2} + \frac{\sqrt{3}}{2}i) = \boxed{\frac{4}{3} + \frac{4\sqrt{3}}{3}i}$$

11) _____

Use De Moivre's theorem to simplify the expression. Write the answer in $a + bi$ form.

12) $(2(\cos 45^\circ + i \sin 45^\circ))^3$

$$2^3 (\cos(3 \cdot 45^\circ) + i \sin(3 \cdot 45^\circ))$$

$$8 (\cos(135^\circ) + i \sin(135^\circ)) = 8(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i) = \boxed{-4\sqrt{2} + 4\sqrt{2}i}$$

12) _____

13) $(2 - 2i)^5$
in 4th Quadrant

$$r = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \tan^{-1}(-\frac{2}{2}) = -45^\circ \text{ or } 315^\circ$$

$$(2\sqrt{2})^5 (\cos(5 \cdot 315^\circ) + i \sin(5 \cdot 315^\circ))$$

$$128\sqrt{2} (\cos(1575^\circ) + i \sin(1575^\circ)) = 128\sqrt{2} (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i) = \boxed{-128 + 128i}$$

13) _____

Find the indicated roots. Write the answer in trigonometric form.

14) Cube roots of $125(\cos 312^\circ + i \sin 312^\circ)$ $n = 3$

$$\sqrt[3]{125} = 5 \quad \frac{312^\circ}{3} = 104^\circ \quad \frac{360^\circ}{3} = 120^\circ$$

14) _____

$$z_1 = \boxed{5(\cos 104^\circ + i \sin 104^\circ)}$$

$$z_2 = 5(\cos(104^\circ + 120^\circ) + i \sin(104^\circ + 120^\circ)) = \boxed{5(\cos(224^\circ) + i \sin(224^\circ))}$$

$$z_3 = 5(\cos(224^\circ + 120^\circ) + i \sin(224^\circ + 120^\circ))$$

$$\boxed{z_3 = 5(\cos(344^\circ) + i \sin(344^\circ))}$$

Find all specified roots.

15) Cube roots of 8

$$n=3$$

$$\theta = 0^\circ$$

$$\frac{360^\circ}{3} = 120^\circ$$

15) _____

$$\sqrt[3]{8} = 2$$

$$z_1 = 2(\cos 0^\circ + i \sin 0^\circ) = 2(1 + i0) = \boxed{2}$$

$$z_2 = 2(\cos 120^\circ + i \sin 120^\circ) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \boxed{-1 + \sqrt{3}i}$$

$$z_3 = 2(\cos 240^\circ + i \sin 240^\circ) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \boxed{-1 - \sqrt{3}i}$$

Solve the equation. Write the answer in a + bi form.

16) $x^2 + 8 = 0$

$$x^2 = -8 \quad \text{find square roots of } -8$$

$$r=8$$

$$\theta = 180^\circ$$

$$\frac{\theta}{2} = 90^\circ$$

$$\frac{360^\circ}{2} = 180^\circ$$

16) _____

$$z_1 = \sqrt{8}(\cos 90^\circ + i \sin 90^\circ) = \sqrt{8}(0 + i) = \sqrt{8}i = \boxed{2\sqrt{2}i}$$

$$z_2 = \sqrt{8}(\cos 270^\circ + i \sin 270^\circ) = \sqrt{8}(0 - i) = -\sqrt{8}i = \boxed{-2\sqrt{2}i}$$

Convert the rectangular coordinates to polar coordinates, using radian measure for the angle.

17) (8, -8)

$$r = \sqrt{(8)^2 + (-8)^2} = \sqrt{128} = 8\sqrt{2}$$

17) _____

4th
Quadr.

$$\theta = \tan^{-1}\left(-\frac{8}{8}\right) = -\frac{\pi}{4}$$

$$\boxed{(8\sqrt{2}, 7\pi/4)}$$

Convert to rectangular coordinates.

18) $\left(-4, -\frac{\pi}{3}\right)$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$x = -4\left(\cos\left(-\frac{\pi}{3}\right)\right) = -4\left(\frac{1}{2}\right) = -2$$

$$y = -4\left(\sin\left(-\frac{\pi}{3}\right)\right) = -4\left(-\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

$$\boxed{(-2, 2\sqrt{3})}$$

18) _____

For the point given in rectangular coordinates, find equivalent polar coordinates (r, θ) for $r > 0$ and $0^\circ \leq \theta < 360^\circ$.

19) $(-4, 4\sqrt{3})$

$$r = \sqrt{(-4)^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = \sqrt{64} = 8$$

19) _____

2nd
Quadr.

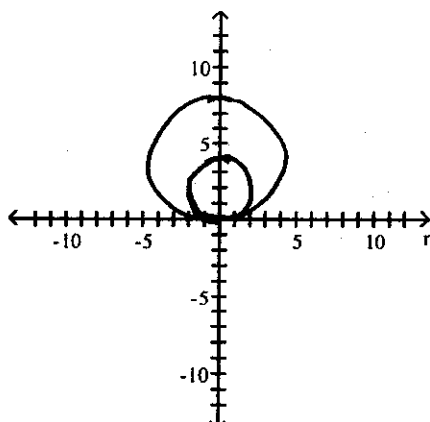
$$\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right) = 120^\circ$$

$$\boxed{(8, 120^\circ)}$$

Graph the polar equation.

20) $r = 2(1 + 3 \sin \theta)$

20) _____



For the given polar equation, write an equivalent rectangular equation.

21) $r = 10 \sin \theta$

$$r \cdot r = 10 \cdot r \sin \theta$$

$$r^2 = 10 r \sin \theta$$

$$\boxed{x^2 + y^2 = 10y}$$

21) _____

Using the pair of parametric equations, find the values of A and B in the table.

22) $x = 3t + 7, y = t + 8, \text{ for } 0 \leq t \leq 7$

t	x	y
0	7	7
A	19	
2		B

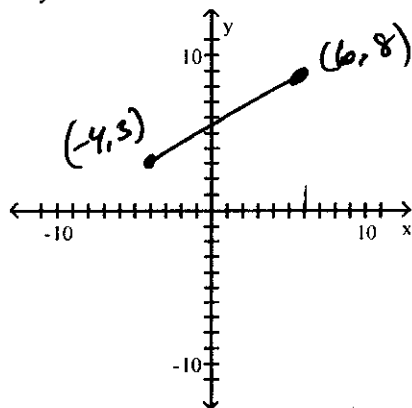
For A
 $19 = 3A + 7$
 $-7 \quad -7$
 $12 = 3A$
 $\boxed{A = 4}$

For B
 $B = 2 + 8$
 $\boxed{B = 10}$

22) _____

Graph the pair of parametric equations in the rectangular coordinate system.

23) $x = 2t, y = t + 5; -2 \leq t \leq 3$



t	x	y
-2	-4	3
-1	-2	4
0	0	5
1	2	6
2	4	7
3	6	8

23) _____

Eliminate the parameter of the pair of parametric equations.

24) $x = t - 3, y = t^2 + 5$

$$x + 3 = t$$

$$y = (x + 3)^2 + 5$$

$$y = x^2 + 6x + 9 + 5$$

$$\boxed{y = x^2 + 6x + 14}$$

24) _____

Solve the problem.

25) A projectile is fired with an initial velocity of 300 feet per second at an angle of 45° with the horizontal. To the nearest foot, find the maximum altitude of the projectile. The parametric equations for the path of the projectile are

$$x = (300 \cos 45^\circ)t, \text{ and}$$

$$y = (300 \sin 45^\circ)t - 16t^2.$$

y - gives the height at time t.
 use the y equation to find max height.

Graph: $y = (300 \sin(45^\circ))x - 16x^2$, on calculator. (degree mode)
 Calculate the maximum height use calc. button...

$$\boxed{y = 703 \text{ ft}}$$

25) _____